

# Similarities and Differences in 2D ‘metallicity’ induced by temperature and parallel magnetic field: To screen or not to screen

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We compare the effects of temperature and parallel magnetic field on the two-dimensional metallic behavior within the unified model of temperature and field dependent effective disorder arising from the screened charged impurity scattering. We find, consistent with experimental observations, that the temperature and field dependence of resistivity should be qualitatively similar in n-Si MOSFETs and different in n-type GaAs 2D systems, with the p-type 2D GaAs system being somewhat intermediate. Based on our calculated results we critically comment on the expected similarities and differences between temperature and field dependent carrier transport properties in various dilute 2D semiconductor systems.

## I. INTRODUCTION

It has been experimentally well-established over the last ten years that low-density and low-disorder two-dimensional (2D) electron systems often exhibit unusually strong temperature ( $T$ ) and in-plane (parallel to the 2D layer) magnetic field ( $B_{\parallel}$ ) dependence in its low-temperature ( $T \sim 1K$ ) resistivity,  $\rho$  [1]. This anomalously strong temperature and field dependence of 2D resistivity in the putative metallic phase, i.e., for carrier densities  $n > n_c$  where  $n_c$  is the critical density for the 2D metal-insulator transition (MIT), i.e. the system is an effective 2D metal for  $n > n_c$  and a strongly localized insulator for  $n < n_c$ , has attracted a great deal of attention because no such behavior is seen in normal 3D metals where the low-temperature  $\rho$  typically saturates (the so-called Bloch-Grüneisen behavior) manifesting little temperature and/or magnetic field dependence. The purpose of this paper is to theoretically explore the connection, if any, between  $\rho(T)$  and  $\rho(B_{\parallel})$ , i.e., the relationship between temperature and magnetic field dependence of 2D resistivity, and to investigate whether both originate from a single physical mechanism.

The connection between  $\rho(T)$  and  $\rho(B_{\parallel})$  has been emphasized in the literature, for example, Pudalov *et al.* [2] carried out a detailed experimental analysis comparing effects of temperature and parallel field on the strength of the ‘metallic’ resistivity in Si MOSFETs. In particular, Pudalov *et al.* found that “the data for magneto- and temperature dependence of the resistivity of Si MOS samples in parallel field may be well-described by a simple mechanism of the magnetic field dependent disorder” [2]. We argue in this paper through explicit calculations of  $\rho(T, B_{\parallel})$  that this simple single mechanism giving rise to strong temperature and parallel-field dependence of 2D resistivity is electronic screening of the quenched disorder which, for low-density and low-temperature semiconductor structures of relevance in the 2D MIT problem, arises from the randomly distributed (unintentional) background charged impurity

centers providing a long-range *bare* Coulombic disorder with the main resistive low-temperature scattering mechanism being carrier scattering by the effective screened Coulombic impurity disorder.

The physical origin of the anomalously strong temperature and magnetic field dependence is the relatively low carrier density involved in the 2D “metallic” phase, which makes it possible for the relevant dimensionless temperature,  $t \equiv T/T_F \propto n^{-1}$ , and magnetic field,  $b \equiv B_{\parallel}/B_s \propto n^{-1}$ , parameters to be large (of order unity) in the 0.1 – 1 K temperature and 1 – 10 T magnetic field range in which the 2D MIT experiments are typically carried out (where  $T_F \equiv E_F/k_B$ , the Fermi temperature, and  $B_s = 2E_F/g\mu_B$ , the spin polarization field at which the applied field completely spin-polarizes the 2D system, with  $k_B$ ,  $E_F$ ,  $g$ ,  $\mu_B$  being respectively the Boltzmann constant, the Fermi energy, the effective carrier Landé  $g$ -factor, and Bohr magneton). Such large values of  $t$  and  $b$  in 2D semiconductor systems (by contrast, the typical value of  $t$  and  $b$  in 3D metals is  $10^{-4}$ ) make the screened effective disorder strongly temperature and magnetic field dependent, leading to unusually strong temperature and magnetic field dependence of  $\rho(T, B_{\parallel})$ .

This ‘simple’ single mechanism of screening (i.e. strongly temperature and field dependent effective disorder) also immediately explains why one needs rather high-quality low-disorder 2D systems with high mobilities for the manifestation of 2D MIT – for highly disordered systems the typical critical density  $n_c$  for the 2D MIT is rather high, making the effective metallic phase exist only at high carrier densities and therefore the parameter  $t$  (and  $b$ ) cannot become very large (since  $t, b \propto n^{-1}$ ) until one reaches higher temperatures where phonon scattering becomes effective. But, screening being operational at all carrier densities, clear signatures of strong temperature and magnetic field dependence of  $\rho(T, B_{\parallel})$  should exist even at reasonably high densities except that the total variation  $\Delta\rho$  due to increasing  $T$  and/or  $B_{\parallel}$  cannot be very large in magnitude at higher

densities since the dimensionless parameters  $t$  and  $b$  cannot really become large at high carrier densities. Indeed, such signatures of 2D “metallicity” in the temperature dependent resistivity are seen in high-quality Si MOS systems at higher carrier densities. The new feature discovered by Kravchenko *et al.* [3] is the continuous enhancement of the temperature dependence, starting deep in the high-density “metallic” phase, as the density is lowered staying within the metallic phase, arising from the increasing value of  $t \equiv T/T_F$  until the critical density for the insulating phase is reached. Similarly, the parallel field dependence of 2D resistivity also persists well into high carrier densities ( $n \gg n_c$ ) deep into the metallic phase except that the effect becomes quantitatively weaker at higher densities since the dimensionless  $b \equiv B_{\parallel}/B_s \propto n^{-1}$  cannot become large at high densities. It is the continuous density induced evolution of  $\rho(T)$  and  $\rho(B_{\parallel})$ , as effective  $t$  and  $b$  acquire large values with decreasing carrier density, which signals the fact that the physical mechanism responsible for the anomalously strong temperature/field dependence of  $\rho(T, B_{\parallel})$  at low densities is already operational at high carrier densities.

The rest of this paper is organized as follows. In sec. II we provide the theory; in sec. III we give our calculated results for the magnetoresistance  $\rho(B_{\parallel})$  comparing it with  $\rho(T)$  for Si MOSFETs, 2D n-GaAs and 2D p-GaAs heterostructure systems; in sec. IV we discuss our results emphasizing the relative importance of spin-polarization induced magneto-screening suppression and magneto-orbital coupling effects on the  $\rho(B_{\parallel})$  in different 2D systems. We conclude in sec. V emphasizing our new results and qualitative conclusion.

## II. THEORY

To understand the  $\rho(T, B_{\parallel})$  behavior quantitatively we start with the Drude-Boltzmann semiclassical formula for 2D transport limited by screened charged impurity scattering [4]:

$$\rho^{-1} \equiv \sigma = ne^2 \langle \tau \rangle / m, \quad (1)$$

where

$$\langle \tau \rangle = \frac{\int \frac{d^2 k}{(2\pi)^2} \epsilon_{\mathbf{k}} \tau(\epsilon_{\mathbf{k}}) \left( -\frac{\partial f(\epsilon_{\mathbf{k}})}{\partial \epsilon_{\mathbf{k}}} \right)}{\int \frac{d^2 k}{(2\pi)^2} \epsilon_{\mathbf{k}} \left( -\frac{\partial f(\epsilon_{\mathbf{k}})}{\partial \epsilon_{\mathbf{k}}} \right)}, \quad (2)$$

with the momentum-dependent transport relaxation time  $\tau(\epsilon_{\mathbf{k}})$  given by the leading-order approximation:

$$\frac{1}{\tau(\epsilon_{\mathbf{k}})} = \frac{2\pi}{\hbar} \int \frac{d^2 \mathbf{k}'}{(2\pi)^2} \int N_i(z) dz |u_i(\mathbf{k} - \mathbf{k}'; z)|^2 (1 - \cos \theta_{\mathbf{k}\mathbf{k}'}) \delta(\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'}). \quad (3)$$

In Eqs. (1)-(3)  $\sigma$  and  $\tau$  are respectively the 2D conductivity and relaxation time with  $\epsilon_{\mathbf{k}}$ ,  $f(\epsilon_{\mathbf{k}})$  as the electron energy [ $\epsilon_{\mathbf{k}} = \hbar^2 k^2 / 2m$ ] and the Fermi distribution function respectively. Eqs. (1) and (2), defining the conductivity and the thermally averaged relaxation time respectively, are standard results of the Boltzmann transport theory whereas Eq. (3) defining the momentum-dependent relaxation time explicitly depends on the three-dimensional distribution of charged impurities,  $N_i(z)$ . The most important quantity for our consideration is the effective disorder scattering potential  $u_i(\mathbf{q}; z)$  in Eq. (3), which, for charged impurity scattering, should be the screened charged impurity scattering:

$$u_i(\mathbf{q}; z) \equiv v_i(\mathbf{q}; z) / \varepsilon(q), \quad (4)$$

where  $v_i$  is the bare (Coulomb) electron-impurity scattering potential, and  $\varepsilon(q)$  is the static RPA dielectric function. The strong variation in  $\rho(T; B_{\parallel})$  with  $T$  and  $B_{\parallel}$  arises in this simple physical picture from the strong variation in the effective disorder potential  $u_i$  due to the variation in the dielectric (i.e. the screening) function  $\varepsilon(q)$  imposed by large variations in the dimensionless temperature ( $t \equiv T/T_F$ ) and magnetic field ( $b \equiv B_{\parallel}/B_s$ ). Note that at  $T = 0$ , we have  $\varepsilon(q) = 1 + q_{TF}/q$  in 2D for  $q \leq 2k_F$ , and  $2k_F$  scattering,  $|\mathbf{k} - \mathbf{k}'| = 2k_F$ , is the most important resistive scattering process so that  $q_{TF}/2k_F \propto n^{-1/2}$  is the nominal control parameter for the strength of metallicity. The Thomas-Fermi screening wave vector  $q_{TF}$  and the Fermi wave vector in 2D systems are given respectively by the formula  $q_{TF} = g_d m e^2 / (\kappa \hbar^2)$  and  $k_F = (4\pi n / g_d)^{1/2}$ , where  $n$  is the 2D carrier density,  $\kappa$  is the background lattice dielectric constant, and  $g_d$  is the degeneracy factor including the spin degeneracy.

Both of these situations, screening-induced variations in  $\rho(T)$  [4] and in  $\rho(B_{\parallel})$  [5], have already been separately theoretically discussed in the 2D MIT literature, with considerable success in qualitatively explaining the experimental data. The purpose of the current paper is to critically analyze the connection and relationship between the overall strengths (“metallicity”) of temperature and field dependence of resistivity. In particular, the important effect of the in-plane field on screening is through spin-polarization of the carriers which could continuously change the spin-degeneracy factor from 2 (at  $B_{\parallel} = 0$ ) to 1 (at  $B_{\parallel} = B_s$ , when the carriers are completely spin-polarized), so that the Thomas-Fermi screening wave vector  $q_{TF}$ , which is proportional to the density of states and hence to the spin degeneracy factor  $g_s$ , decreases by a factor of 2 as  $B_{\parallel}$  goes from  $B_{\parallel} = 0$  to  $B_{\parallel} \geq B_s$ . In the strong screening situation (i.e., for  $q_{TF} \gg 2k_F$ , where  $k_F \propto \sqrt{n}$  is the Fermi wave vector), which typically applies to Si MOSFETs (but not to 2D n-GaAs systems), this would then imply that  $\rho(B_{\parallel})/\rho(B_{\parallel} = 0) \equiv r_b(B_{\parallel})$  could have an absolute maximum value of 4 in the ideal situation with  $r_b(B_{\parallel})$  being a constant for  $B_{\parallel} > B_s$ , and  $r_b(B_{\parallel})$  rising continuously

in the  $0 \leq B_{\parallel} \leq B_s$  regime. Si MOSFETs seem to obey this behavior quite well since  $q_{TF} \gg 2k_F$  strong screening condition is well satisfied in Si MOSFETs at all experimental “metallic” densities ( $n \geq 10^{11} \text{cm}^{-2}$ ).

In discussing  $\rho(T)$ , however, it is not possible to obtain such a simple maximum ideal value for  $\rho(T)/\rho(T=0) \equiv r_t(T)$  because thermal effects on screening do *not* saturate at some optimum temperature (unlike the magnetic field induced spin polarization dependent screening effect which does saturate at  $B_{\parallel} = B_s$ ). In principle, screening vanishes in the extreme high temperature limit,  $T \gg T_F$ , leading to an absolute upper limit on the temperature induced resistivity enhancement of  $\rho(T \gg T_F)/\rho(T=0) \equiv r_t(T \gg T_F) \approx (q_{TF}/2k_F)^2$  through the screening effect (in the strong screening  $q_{TF} \gg 2k_F$  regime). We emphasize that, unlike  $r_s(B_{\parallel})$  which actually does have a reasonable ideal maximum of 4 for magnetic field induced resistivity enhancement, the *theoretical* ideal maximum  $r_t(T \gg T_F) = (q_{TF}/2k_F)^2$  is *not* a reasonable value for the practical observable temperature induced maximum enhancement of resistivity. This is because at high temperatures ( $T \sim T_F$ ) various other effects (e.g. smearing of the Fermi surface, thermal averaging, etc.) become operational and  $\rho(T)$  *actually* decreases [4] with increasing temperature in the  $T \geq T_F$  regime. (Also, at higher temperatures phonon scattering invariably becomes significant, leading eventually to a monotonically increasing  $\rho(T)$  with temperature.) Nevertheless,  $r_t(T \gg T_F) = (q_{TF}/2k_F)^2$  is *not* completely wrong as a crude qualitative measure of the theoretical upper limit on the temperature induced maximal enhancement of  $\rho(T)$ . It is easy to derive a simple formula connecting  $\tilde{r}_b \equiv r_b(B_{\parallel} = B_s)$  and  $\tilde{r}_t \equiv r_t(T/T_F \rightarrow \infty)$  *using simply the screening considerations* given above (which will obviously not be correct, but may be used for qualitative considerations):  $\tilde{r}_b \equiv 4\tilde{r}_t[\tilde{r}_t + 2\sqrt{\tilde{r}_t} + 1]^{-1}$ . We caution that this formula typically strongly overestimates  $r_t$ . The basic picture that emerges from the above qualitative considerations is that  $\rho(T)$  and  $\rho(B_{\parallel})$  should, in general, have “similar” anomalously strong metallic behavior – in particular, in the strong screening situation ( $q_{TF} \gg 2k_F$ ) both should show anomalously large increase in the resistivity as  $t$  and  $b$  increase from zero to order unity (which is made possible by low Fermi energy at low carrier densities so that  $t \equiv T/T_F$  and  $b \equiv B_{\parallel}/B_s$  can be large). Note that at this stage it is not possible to make a more direct quantitative connection (except for the simple formula which overestimates the temperature dependence) between  $r_t$  and  $r_b$  except to say that they should both be large or both be small. Note also that we have only discussed so far the spin-polarization induced magneto-screening effect on  $\rho(B_{\parallel})$  leaving out other possible corrections (e.g. magneto-orbital coupling) which could be quantitatively important.

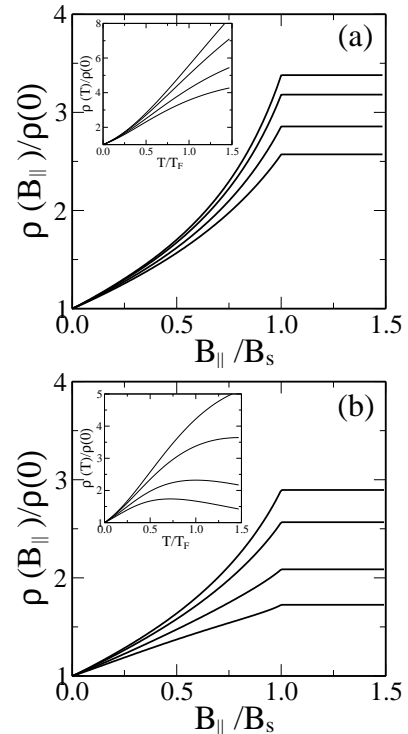


FIG. 1. The calculated  $\rho(B_{\parallel})$  in Si MOSFETs (a) for ideal 2D system and (b) for quasi-2D system at  $T = 0$  and for various densities  $n = 1.0, 2.0, 5.0, 10.0 \times 10^{11} \text{cm}^{-2}$  (from top to bottom). The insets show  $\rho(T)$  at  $B_{\parallel} = 0$  and for the same systems as the main figures.

### III. RESULTS

We now apply these concrete considerations to n-Si MOSFET and n-GaAs 2D systems, which are representative examples of strong-screening (n-Si MOSFET) and weak-screening (n-GaAs) 2D systems, respectively. We also show calculated results for holes in the 2D p-GaAs system which is somewhat intermediate between Si MOSFETs and n-GaAs systems in terms of screening properties.

In Fig. 1 we show our full numerical Drude-Boltzmann calculations for  $\rho(T)$  and  $\rho(B_{\parallel})$  in Si MOSFETs assuming ideal (zero-width) 2D and (finite-width) quasi-2D systems with the charged impurity scatterers located randomly at the Si-SiO<sub>2</sub> interface. It is clear that for Si MOSFETs  $\rho(T)$  and  $\rho(B_{\parallel})$  manifest qualitatively similar “anomalous” metallic behavior over a large range of density, temperature, and magnetic field values, both for the strictly 2D and for the realistic quasi-2D models. This qualitative similarity, already noted in the corresponding experimental situation by Pudalov *et al.* [2], arises entirely from the strong-screening nature ( $q_{TF} \gg 2k_F$ ) of Si MOSFETs due to its rather large effective mass and valley degeneracy of 2 which makes  $q_{TF}/2k_F|_{\text{Si}} \approx 16/\sqrt{\tilde{n}}$  where  $\tilde{n}$  is the carrier density measured in units of  $10^{11} \text{cm}^{-2}$ . In the usual density range of our interest,

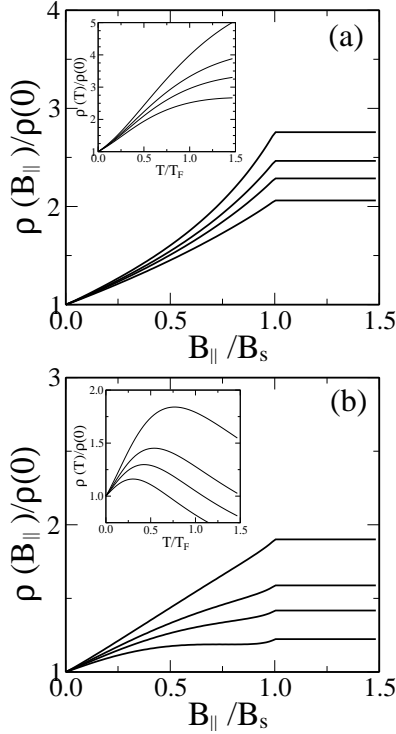


FIG. 2. The calculated  $\rho(B_{||})$  in n-GaAs (a) for ideal 2D system and (b) for quasi-2D system at  $T = 0$  and for various densities  $n = 0.4, 0.8, 1.2, 2.0 \times 10^{10} \text{ cm}^{-2}$  (from top to bottom). The insets show  $\rho(T)$  at  $B_{||} = 0$  and for the same systems as the main figures.

therefore, the strong screening condition  $q_{TF} \gg 2k_F$  is always satisfied for Si MOSFETs leading to qualitatively similar behavior in  $\rho(T)$  and  $\rho(B_{||})$ , as shown in Fig. 1 and as observed experimentally.

Now we consider the opposite extreme of electrons in 2D n-GaAs heterostructure system where screening is generally weak and the  $q_{TF} \gg 2k_F$  strong-screening condition can only be satisfied at very low carrier densities:  $q_{TF}/2k_F|_{\text{n-GaAs}} \approx 1.3/\sqrt{n}$ . We therefore expect rather weak temperature and field dependence of resistivity in 2D n-GaAs system, which is even further exacerbated by the relatively large values of the Fermi energy (due to the small effective mass) which make the relative values of the dimensionless temperature ( $t \equiv T/T_F$ ) and magnetic field ( $b \equiv B_{||}/B_s$ ) parameters rather low in the experiments. In Fig. 2 we show our 2D n-GaAs calculation for  $\rho(T)$  and  $\rho(B_{||})$ , again in the ideal 2D and realistic quasi-2D approximations, taking into account screened charged impurity scattering at the interface. As expected (due to the relatively low value of  $q_{TF}/2k_F$  in n-GaAs), both temperature and magnetic field dependence are weak in Fig. 2, again manifesting the underlying connection between the field and temperature dependence of the effective disorder arising from the temperature and magnetic field dependent screening of charged impurity screening.

Comparing Figs. 1 and 2 we tentatively conclude that

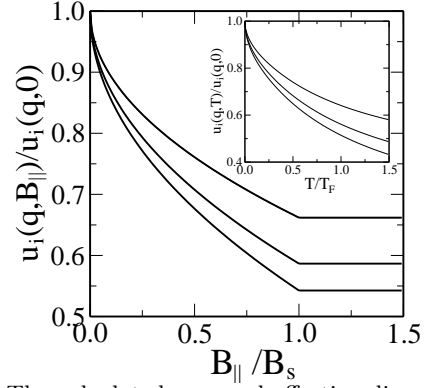


FIG. 3. The calculated screened effective disorder scattering potential  $u_i(q, B_{||})/u_i(q, 0)$  at  $q = 2k_F$  and  $T=0$  as a function of the magnetic field for various screening parameter  $q_{TF}/2k_F = 5, 10, 20$  (from top to bottom). The inset shows  $u_i(q, T)/u_i(q, 0)$  at  $q = 2k_F$  and  $B=0$  as a function of temperature.

there is a compelling theoretical qualitative similarity between  $\rho(T)$  and  $\rho(B_{||})$  for charged impurity scattering limited 2D transport arising essentially entirely from the “similar” temperature and field induced weakening of screening: strong (weak) variation in  $\rho(T)$  implies corresponding strong (weak) variation in  $\rho(B_{||})$  and vice versa as is manifest in Fig. 1(2) for n-Si MOSFET (n-GaAs). To make this point more explicit we show in Fig. 3 our calculated screened effective impurity disorder  $|u_i(q = 2k_F)|$  as a function of the dimensionless magnetic field and temperature parameters for a number of different values of the screening parameter  $q_{TF}/2k_F$ . It is obvious that for large values of  $q_{TF}/2k_F$ , the effective disorder varies strongly (and similarly) with  $t$  and  $b$ , leading to qualitatively similar strong variation in  $\rho(T)$  and  $\rho(B_{||})$ .

The above-discussed qualitative theoretical similarity in  $\rho(T)$  and  $\rho(B_{||})$  is entirely consistent with the available experimental data in Si MOSFETs, where both  $\rho(B_{||})$  and  $\rho(T)$  show anomalously strong variations at low carrier densities. But, in 2D n-GaAs system the existing experimental data on  $\rho(T)$  and  $\rho(B_{||})$  are in *striking disagreement* with the screening theory predictions shown in our Fig. 2. In particular, the experimental  $\rho(B_{||})$  shows a strong variation with the magnetic field whereas the experimental  $\rho(T)$  shows a rather weak variation [6,7], which obviously is inconsistent with the simple screening picture. While a part of this discrepancy can be explained by the experimental value of the dimensionless temperature parameter (i.e.,  $t \equiv T/T_F$ ) being smaller than the corresponding magnetic field parameter (i.e.,  $b \equiv B_{||}/B_s$ ) because phonon scattering becomes important at higher temperatures in GaAs, the overall qualitative disagreement between experiment and theory in 2D n-GaAs system is inexplicable within the screening theory. Some other ingredient of physics is missing in

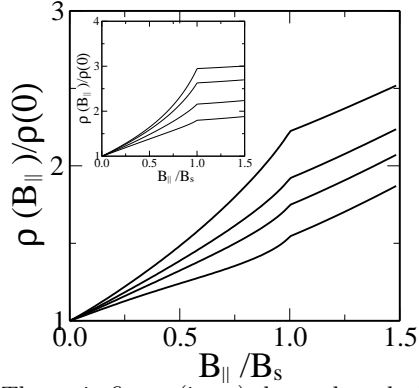


FIG. 4. The main figure (inset) shows the calculated  $\rho(B_{||})$  in the realistic quasi-2D n-GaAs (Si MOS) system including both magneto-spin polarization and magneto-orbital effects for various densities  $n = 0.4, 0.8, 1.2, 2.0 \times 10^{10} \text{ cm}^{-2}$  ( $n = 1.0, 2.0, 5.0, 10.0 \times 10^{11} \text{ cm}^{-2}$ ), from top to bottom.

the theory as far as the  $\rho(B_{||})$  behavior is concerned in the 2D GaAs system – the  $\rho(T)$  behavior is qualitatively well-explained [7] by the screening theory results shown in Fig. 2. The missing piece of physics is the magneto-orbital effect [8] which affects the 2D GaAs system much more strongly than the 2D Si system since the 2D confinement is much weaker in GaAs by virtue of its small effective mass and low density. When the quasi-2D layer width is larger than the magnetic length associated with the parallel field, the 2D orbital dynamics is strongly affected by the magnetic field with a strong increase in the field dependent 2D effective mass as well as by intersubband scattering [8]. We refer to our earlier work [8] for details on the magneto-orbital effects.

In Fig. 4 we show our calculated  $\rho(B_{||})$  in the realistic quasi-2D n-GaAs (and Si MOS) system including both magneto-spin polarization and magneto-orbital effects [8]. It is clear that both magneto-spin polarization and magneto-orbital effects are essential in understanding the GaAs data whereas in Si MOSFETs the magneto-orbital effects are small due to the rather tight confinement of the quasi-2D electron wavefunction. We note that  $\rho(B_{||} > B_s)$  continues increasing in n-GaAs because of the magneto-orbital effect whereas the spin polarization effect saturates at  $B_{||} = B_s$ . We emphasize that the results of Fig. 4 are in qualitative agreement with the experimental  $\rho(B_{||})$  in ref. [6] whereas the theoretical results with just the spin polarization effects disagree with experiment. One therefore needs to include both magneto-screening and magneto-orbital effects for understanding 2D n-GaAs heterostructure systems whereas experimental data for Si MOSFETs (because of their tight 2D confinement) can be reasonably well-explained without any the magneto-orbital effects.

In this context, it is also interesting to consider 2D holes in the p-GaAs heterostructures because it is a strongly screening ( $q_{TF} \gg 2k_F$ ) system, which also could

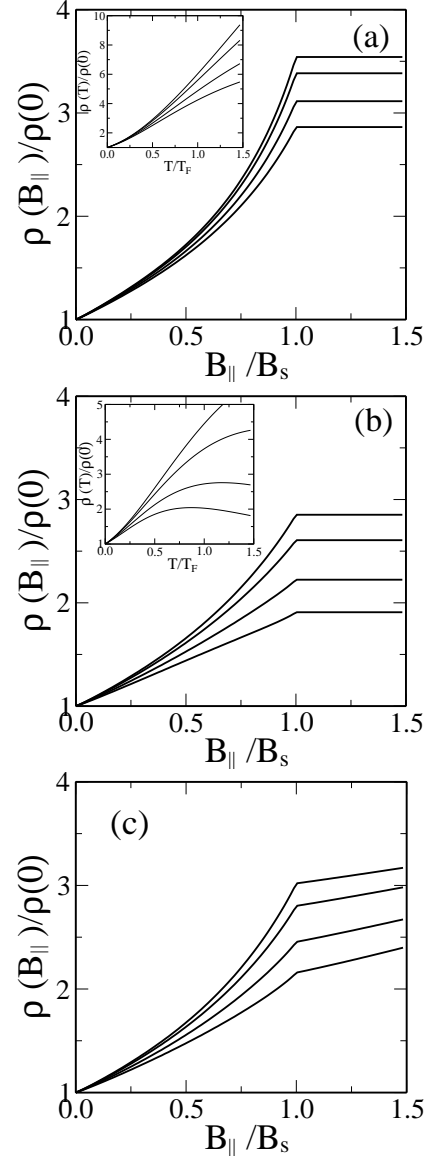


FIG. 5. The calculated  $\rho(B_{||})$  in p-GaAs (a) for ideal 2D system and (b) for quasi-2D system at  $T = 0$  and for various densities  $n = 1.0, 2.0, 5.0, 10.0 \times 10^{10} \text{ cm}^{-2}$  (from top to bottom). The insets show  $\rho(T)$  at  $B_{||} = 0$  and for the same systems as the main figures. In (c) the calculated  $\rho(B_{||})$  in the quasi-2D system including both magneto-spin and magneto-orbital effects is shown for the same densities given in (b).

have substantial magneto-orbital coupling at low carrier densities due to its weak confinement potential leading to the measured magnetic field dependence of the resistivity being quite different from that of Si MOSFET systems. In Fig. 5 we show our calculated  $\rho(T)$  and  $\rho(B_{\parallel})$  of the p-GaAs system in the ideal 2D and realistic quasi-2D systems taking into account screened charged impurity scattering at the interface. As in the n-Si MOSFET systems, the high value of  $q_{TF}/2k_F$  in p-GaAs in the density range of our interest ( $n \geq 10^{10} \text{ cm}^{-2}$ ),  $q_{TF}/2k_F|_{\text{p-GaAs}} \approx 8/\sqrt{\tilde{n}}$  with  $m_h^* = 0.4m_e$ , gives rise to both strong temperature and magnetic field dependence of the resistivity. However, the experimental  $\rho(B_{\parallel})$  of p-GaAs shows no saturation [9] behavior at  $B_{\parallel} \geq B_s$ . Again the increase of  $\rho(B_{\parallel})$  above  $B_s$  in p-GaAs heterostructure arises from the magneto-orbital effect due to the low-density quasi-2D width being larger than the parallel field induced magnetic length. In Fig. 5(c), we show  $\rho(B_{\parallel})$  including both magneto-spin and magneto-orbital effects.

#### IV. DISCUSSION

The main purpose of this article is to point out that the observed “similarity” between the temperature dependence,  $\rho(T)$ , and the parallel field dependence,  $\rho(B_{\parallel})$ , of the 2D resistivity in Si MOSFETs arises from the importance of screening of the long-range Coulombic disorder (associated with random charged impurity centers) at low carrier densities. In particular, increasing  $T/T_F$  reduces screening effects making the effective disorder stronger, leading to a large increase in  $\rho(T)$ . Similarly, increasing the parallel field reduces the screening since the 2D carriers become spin-polarized by the applied field, leading to a decreasing density of states and hence decreasing screening effects. Thus, to the extent screening is the dominant mechanism (i.e. as long as  $q_{TF}/2k_F \gg 1$ ) underlying 2D metallic behavior, one expects a qualitative similarity between  $\rho(T)$  and  $\rho(B_{\parallel})$  as observed empirically in ref. [2] for Si MOSFETs.

Another important point made in this paper is that, although screening is the dominant physical mechanism underlying 2D metallicity, there are several other factors which could affect  $\rho(T)$  and  $\rho(B_{\parallel})$ , leading to considerable difference between the two. In particular,  $\rho(T)$  is affected by phonon scattering and thermal averaging at “higher” values of temperature, and  $\rho(B_{\parallel})$  is affected by the magneto-orbital effect for large quasi-2D widths of the system. While these non-screening effects are quantitatively unimportant in Si MOSFETs in the usual  $n \sim 10^{11} \text{ cm}^{-2}$  experimental density range, the magneto-orbital (as well as phonon scattering and thermal averaging) effects turn out to be important for n-GaAs 2D heterostructures where the 2D confinement potential becomes very weak at low carrier densities leading to strong

magneto-orbital effects. Therefore, in 2D n-GaAs heterostructures (and to a lesser extent in 2D p-GaAs heterostructures), there are significant differences between  $\rho(T)$  and  $\rho(B_{\parallel})$ , compared with the Si MOSFET situation, arising from magneto-orbital coupling.

We note that the magneto-orbital coupling effect, where the applied parallel field couples directly to the carrier orbital dynamics in addition to causing spin-polarization, arises only when the quasi-2D transverse width  $\langle z \rangle$  of the system is larger than the magnetic length  $l_B = \sqrt{\hbar c/eB_{\parallel}}$  associated with the applied parallel field. The magneto-orbital effect may be suppressed in n- and p-GaAs systems by using narrow quantum well systems (with  $\langle z \rangle < l_B$ ) rather than heterostructure systems. In heterostructure, the 2D confinement potential is provided by the self-consistent potential which becomes very shallow at low carrier densities — in particular,  $\langle z \rangle \propto n^{-1/3}$  where  $n$  is the carrier density. In a quantum well, on the other hand, the width ( $a$ ) of the quantum well completely determines the strength of the magneto-orbital coupling, and as long as  $a < l_B$ , magneto-orbital coupling effects can be ignored. This is indeed the experimental finding [10] in p-GaAs quantum wells. We also note that the dimensionless parameters  $B_{\parallel}/B_s \sim B_{\parallel}/n$  and  $\langle z \rangle/l_B \sim B_{\parallel}^{1/2}/n^{1/3}$  respectively control the strength of magneto-screening and magneto-orbital effects in 2D systems. In Si MOSFETs  $\langle z \rangle \leq 50 \text{ \AA}$  in the experimental density range, and the magneto-orbital coupling is unimportant. Finally, we add that the thermal effects on  $\rho(T)$  are controlled by the parameters  $T/T_F$  and  $q_{TF}/2k_F$  as long as phonon scattering effects are unimportant — phonons become important above 5 – 10 K for Si MOSFETs and above 1 – 2 K for n- and p-GaAs 2D systems.

#### V. CONCLUSION

Before concluding it may be worthwhile to emphasize what is new in this paper compared with existing theoretical publications on the temperature and parallel magnetic field dependence of low-density carrier transport in 2D semiconductor structures. We have earlier [4] extensively reported on theoretical results describing the temperature dependence of 2D resistivity in low-density semiconductor structures, and therefore the temperature dependent  $\rho(T)$  results shown as insets of our figures are only provided for the purpose of comparison with the magnetic field dependent resistivity  $\rho(B_{\parallel})$  shown in the main panels of our figures. Our magnetic field dependent 2D resistivity,  $\rho(B_{\parallel})$ , in different systems is new, and shows that, as long as the magneto-orbital coupling of the parallel field to the transverse orbital dynamics of the 2D system can be neglected (i.e. as long as the quasi-2D transverse width of the system is small compared with the magnetic length  $l_B$ ), the suppression of

the screening of the impurity potential by the magnetic field (due to spin polarization) and by the temperature (due to thermal smearing) is very similar. The two systems, the n- and p-type GaAs heterostructures, where the magneto-orbital coupling is strong, the temperature-dependent resistivity  $\rho(T)$  and the field-dependent resistivity  $\rho(B_{\parallel})$  behave differently. It may also be useful to ask about the importance of interaction effects [11] beyond screening in determining the temperature and field dependent 2D resistivity. While interaction effects are undoubtedly important, their quantitative significance is unknown at this stage since the interaction theory appropriate for the realistic long-range impurity potential does not yet exist. The qualitative agreement between our screening theory and experimental results indicate that interaction corrections are of quantitative, but *not* of qualitative importance. One important quantitative effect of interaction is the actual value of the spin-polarization field,  $B_s = 2F_F/g\mu_B \propto n/(gm)$ , which is inversely proportional to the product  $gm$ . It is known [1,6] that  $gm$  is strongly renormalized by interaction effects [12], leading to a large suppression of  $B_s$  compared with the non-interacting value.

We conclude by emphasizing our qualitative findings: (1) in Si MOSFETs, the parallel field dependence of 2D resistivity (at a constant low temperature  $T/T_F \ll 1$ )  $\rho(B_{\parallel})$  and the temperature dependence of the resistivity (at zero parallel field)  $\rho(T)$  should manifest qualitatively “similar” anomalously strong metallic behavior since both are dominated by screening effects; (2) in n-GaAs  $\rho(T)$  should show weak metallicity by virtue of weak screening, but  $\rho(B_{\parallel})$  in low-density heterostructures should show strong  $B_{\parallel}$ -dependence since it is affected both by spin-polarization and magneto-orbital effects; (3) in p-GaAs due to strong screening the resistivity shows both strong temperature and magnetic field dependence, and the magneto-orbital effect is operational in weak confinement heterostructure systems; (4) the “strong” metallicity in 2D semiconductor structures arises from the possibility of having strong screening (i.e. large  $q_{TF}/2k_F$ ) as well as large values of dimensionless temperature ( $T/T_F$ ) and magnetic field ( $B_{\parallel}/B_s$ ), where  $B_s$  is the magnetic field for full spin-polarization of the 2D system; (5) experiments should be carried out in both low-density 2D heterostructures and narrow quantum wells (for n- and p-GaAs systems) in order to separate out magneto-screening and magneto-orbital corrections to  $\rho(B_{\parallel})$ .

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